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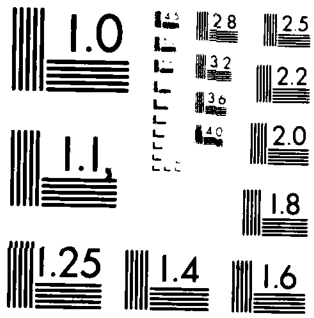
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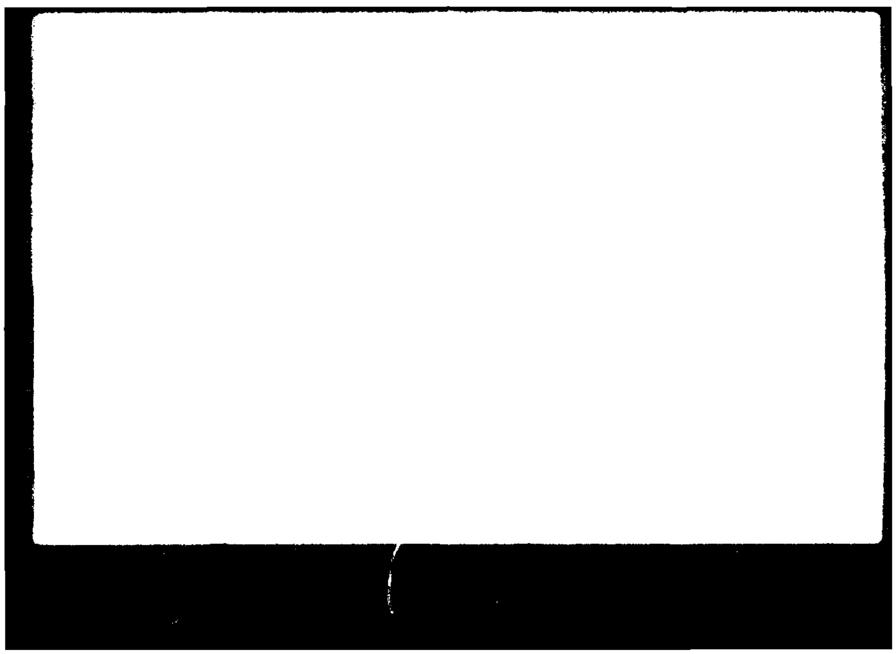

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THREE-DIMENSIONAL ANALYSIS OF COMPOSITE PLATES WITH MATERIAL NONLINEARITY

by

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THREE-DIMENSIONAL ANALYSIS OF COMPOSITE PLATES WITH MATERIAL NONLINEARITY

T. Kuppusamy*, A. Nanda, and J. N. Reddy**

ABSTARCT

A fully three-dimensional analysis of laminated plates with material nonlinearity is presented. The modified Rombor-Osgood relation is used to compute the principal elastic moduli in the plane of the laminae. The elastic-plastic model with Hill's criterion is also used in the study. A shear deformation plate theory is also used to compare the results obtained by the three-dimensional model.

INTRODUCTION

Considerable research effort has gone in the past into the analysis of laminated composite plates. Much of the earlier research was confined to analyses based on the laminate plate theory of Reissner and Stavsky [1]. Extension of the classical laminate theory to thick plates is due to Yang, Norris, and Stavsky [2]. Closed-form solutions of this theory were presented by Whitney and Pagano[3], and Reddy and Chao [4]. Reddy and his colleagues [5-8] and Spilker[9] presented finite-element analyses based on the shear deformation plate theory. A state of plane stress was assumed in all plate theories, and the individual lamina and hence the laminate is assumed to behave in linear elastic fashion. The finite-element analysis of laminated plates using the three-dimensional elasticity theory has been conducted by Lin [10], Dana [11], Dana and Barker [12], and Kuppusamy and Reddy [13]. The studies in [10-12] were limited to linear elastic analyses while that in [13] was geometricall nonlinear. None of these studies considered material nonlinearity or elasto-plastic behavior in the analyses.

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Griffin, Kamat and Herakovich [14] included material nonlinearity through a elastic-plastic model with strain hardening to study laminated plates subjected to inplane loads. Pifko, Levine, and Armen [15] presented a similar study. Nonlinear material properties were introduced via one-dimensional Ramberg-Osgood representations by Renieri and Herakovich [16]. All of these studies were mainly intended to compare laboratory test data in one-dimensional tension tests with those predicted by FEM/Ramberg-Osgood relation. Plate bending problems were not studied.

The present study involves a fully three-dimensional analysis of composite plates using the finite element method and includes material nonlinearity. Apparently, the present study is the first one to consider the material nonlinearity in three-dimensional finite-element analysis of composite plates. The material nonlinearity is introduced by, 1) uncoupled one-dimensional modified Ramberg-Osgood model; and 2) elastic-plastic model using modified Hill's criterion for anisotropic media. The finite elements developed in [5] and [13] are extended to account for material nonlinearities. Some of the results from the three-dimensional nonlinear analysis are compared with those of shear deformable (2-D) theory.

FINITE ELEMENT FORMULATION

Here we present a displacement finite element model for an elastic body. The kinematic description of an elastic body yields the following equations of equilibrium (in the absence of body forces and moments)

$$\sigma_{ij,j} = 0 \qquad (i,j = 1,2,3) \qquad (1)$$

where σ_{ij} are the stress components, and x_i are the Cartesian coordinates.

The linear stress-strain constitutive relationship is

$$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl} \tag{2}$$

where C_{ijkl} contains the material constants, ϵ_{kl} is the strain tensor.

The field equations are to be adjoined by the boundary conditions specified for the problem. The finite element model associated with the variational form of the equation (see [17]) over an arbitrary element Ω^e of the finite element mesh is given by

$$\int_{\Omega^e} [\delta u_{i,j}\ \sigma_{ij}]\ dx_1\ dx_2\ dx_3 = \int_{\Gamma^e} t_m\ u_m\ ds \tag{3}$$

where δ denotes the variational symbol and t_m is the surface traction on Γ^e.

The displacements u_m are assumed in terms of nodal displacements through interpolation functions. These are expressed in matrix form as:

$$\{u\} = [N]\ \{q\} \tag{4}$$

where $\{q\}$ is the nodal displacement vector and $[N]$ is the interpolation matrix.

For a three-dimensional eight-node brick element, one has

$$N_i = \frac{1}{8}\,(1 + ss_i)\ (1 + \gamma\gamma_i)\ (1 + tt_i) \quad i = 1,\ldots 8 \tag{5}$$

where s, γ, and t are the local coordinates (i.e., in the element).

The strain-displacement relations can be expressed in terms of the column of nodal displacements as

$$[k] = [B]\ \{q\} \tag{6}$$

where [B] is the matrix consisting of the derivatives of the interpolation functions, N_i. Substituting Eqs. (4) and (6) into Eq. (3) and taking the variation, one obtains the element equation

$$[K^e]\ \{q\} = \{R^e\} \tag{7}$$

where

$$[K^e] = \int_{\Omega^e} [B]^T\ [C]\ [B]dv \tag{8}$$

and

$$\{R^e\}\ = \text{applied force vector} = \int_{\Gamma^e} \{t\}[N]^T ds \tag{9}$$

The element equations are assembled in the usual manner to obtain the global equations, which have the general form

$$[K]\ \{Q\} = \{R\} \tag{10}$$

Equation (10) is solved for $\{Q\}$ after introducing the essential boundary conditions. The secondary quantities, i.e., strains and stresses in each element are then computed.

MATERIAL MODELS

Two different nonlinear material models are considered. They are, 1) Modified Ramberg-Osgood model; and 2) elastic-plastic model with modified Hill's criterion for anisotropic media.

Modified Ramberg-Osgood Model

For an anisotropic linear elastic medium, the stress-strain relationship (2) can be written in a matrix form as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{13} & A_{23} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{11}$$

where A_{11} to A_{66} are given in terms of engineering constants E_1, E_2, E_3, ν_{12}, ν_{21}, ν_{13}, ν_{31}, ν_{23}, ν_{32}, G_{12}, G_{13} and G_{23} (see [18]). If the stress-strain relationship is nonlinear, then the coefficients A_{ij} vary during deformation. If it is assumed that the nonlinearity can be represented by uncoupled stress-strain behavior for each of the strain components, then a suitable curve fitted to the experimentally observed stress-strain behavior can serve as the material constitutive law. The Ramberg-Osgood relation used here is one such procedure.

For each of the strain components the stress-strain relationship can be simulated by the Romberg-Osgood relation (see [19,20]) as

$$\varepsilon = \frac{\sigma}{E_i} + \lambda\left(\frac{\sigma}{E_i}\right)^m \tag{12}$$

where ε = strain, σ = stress, E_i = the initial modulus,and λ and m are the parameters defining the curve (see Figure 1).

A modified stress-strain relationship can be derived from the above law as:

$$\sigma = \frac{E_i - E_p}{\left[1 + \left(\frac{E_i - E_p}{\sigma_p}\right)^m\right]^{1/m}} + E_p \tag{13}$$

The tangent modulus can now be defined as

$$E_t = \frac{d\sigma}{d\varepsilon} = \frac{E_i - E_p}{\left[1 + \left(\frac{E_i - E_p}{\sigma_p}\right)^m\right]^{\frac{1+m}{m}}} + E_p \tag{14}$$

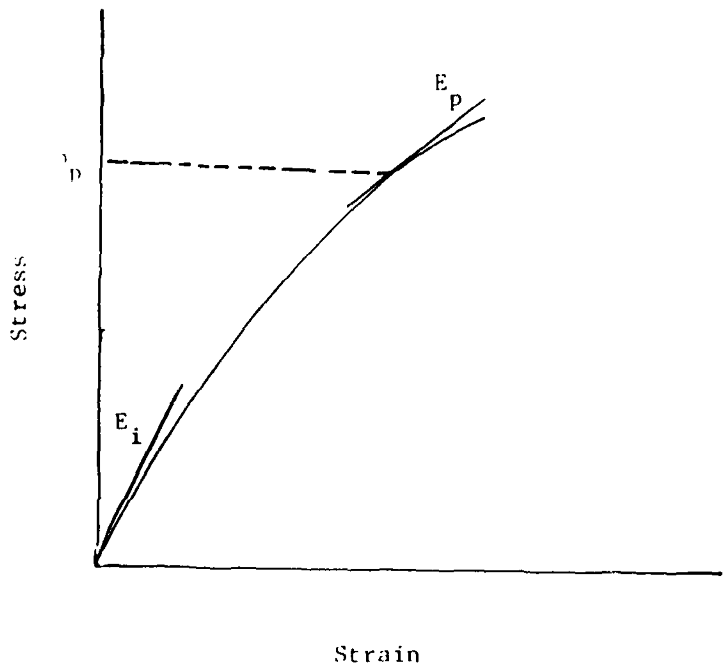


Figure 1. Ramberg-Osgood Model

The above equation is used to calculate the E values at every strain level and update the stiffness matrix in the finite element formulation.

Elastic-Plastic Model

An incremental formulation is used here. It is assumed that

$$\dot{\varepsilon}_i = \dot{\varepsilon}^e_i + \dot{\varepsilon}^p_i \; ; \quad i = 1,2,\ldots,6 \tag{15}$$

where $\dot{\varepsilon}_i$ = incremental total strain, $\dot{\varepsilon}^e_i$ = incremental elastic strain and $\dot{\varepsilon}^p_i$ = incremental plastic strain.

The Hooke's (2) law in incremental form is given by

$$\dot{\sigma}_i = A_{ij}\,\dot{\varepsilon}^e_j \qquad i,j = 1,2,\ldots,6 \tag{16}$$

where A_{ij} = the constitutive matrix given in Eq.(11).

As per the modified Hill's criterion for an orthotropic medium, the following yield function is used [26]:

$$2f(\sigma_i) = F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L(\sigma_5)^2 + 2M(\sigma_6)^2 + 2N(\sigma_4)^2 = 1 \tag{17}$$

where $f(\sigma_i)$ = yield function, and material parameters F, G, H, L, M, and N are defined in terms of yield strengths as:

$$\begin{aligned} 2F &= \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \\ 2G &= \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2} \\ 2H &= \frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2} \\ 2L &= \frac{1}{R^2}, \quad 2M = \frac{1}{S^2}, \quad 2N = \frac{1}{T^2} \end{aligned} \tag{18}$$

where X,Y, and Z are the normal yield strengths in the material principal directions and R,S and T are the shear yield strengths.

The normality condition is:

$$\dot{\varepsilon}^p_i = \bar{\lambda} \frac{\partial f}{\partial \sigma_i} \qquad i = 1,2,\ldots,6 \tag{19}$$

where $\bar{\lambda}$ can be evaluated from the consistency condition give by

$$df = \frac{\partial f}{\partial \sigma_i} \dot{\sigma}_i = 0 \qquad i = 1,2,\ldots,6 \tag{20}$$

It can be obtained that

$$\bar{\lambda} = \frac{1}{K} D_i f_i \qquad i = 1,2\ldots,6 \tag{21}$$

where $K = D_i f_i$, $D_i = f_j A_{ij}$, and $f_i = \frac{\partial f}{\partial \sigma_i}$.

Substituting Eq. (16), (19) and (21) into Eq. (15), one obtains the constitutive equation

$$\dot{\sigma}_i = C_{ij} \dot{\varepsilon}_j \qquad i,j = 1,2,\ldots,6 \tag{22}$$

where C_{ij} is the elastic-plastic matrix which should be updated for nonlinear analysis at each step.

The nonlinear analysis is carried out by the incremental method combined with iterative correction for the load vector at every load step (see [27]). The following iterative process is adopted:

a. the load increment is applied and the elastic stress and strain increments are obtained.

b. The yield function $f(\sigma)$ is calculated for each element. For the elements where $f > 0$, the elasto-plastic strain increments are calculated using the elastic-plastic matrix [see Eq.(22)]. The stresses are modified in the proportion of the strain

increment to artificially make $f > 0$. The differences in the stresses are treated as the initial stresses.

c. The residual load vector is computed from the initial stresses, and step (b) is repeated. The iteration is carried out the residual vector becomes very small (say one percent of the load vector in the preceding iteration). Once convergence is achieved, the next load increment is added and the whole iteration procedure is repeated.

RESULTS

In order to validate the elastic-plastic model used here, the problem of an isotropic plate with a notch (see Fig. 2) is solved by using the formulation developed here for the three-dimensional analysis. The notch stress vs. displacement is shown in Fig. 1. The yield strength of the material is 30 kg/mm^2 and $E = 0.2 \times 10^5 kg/mm^2$ and $\nu = 0.3$. The present three-dimensional analysis consists of 72 brick elements; the maximum notch stress at the end of six, seven, and eight iterations is, respectively, 19, 21 and 23 kg/mm^2. The convergence trend indicates that the maximum stress value will be lower than 23 kg/mm^2. Results of a two-dimensional (plane stress) analysis of the same problem is available in Reference 23. The maximum notch stress was 19 kg/mm^2 and this was obtained by using 245 two-dimensional plane elements and 51 iterations. From the classical slip line solution of this problem the maximum notch stress is 18 kg/mm^2. Considering the fact that coarse mesh and fewer iterations (8 iterations) are used in the present study, and the present study is based on the three-dimensional formulation of the problem, one can conclude that the results obtained here are reasonably good.

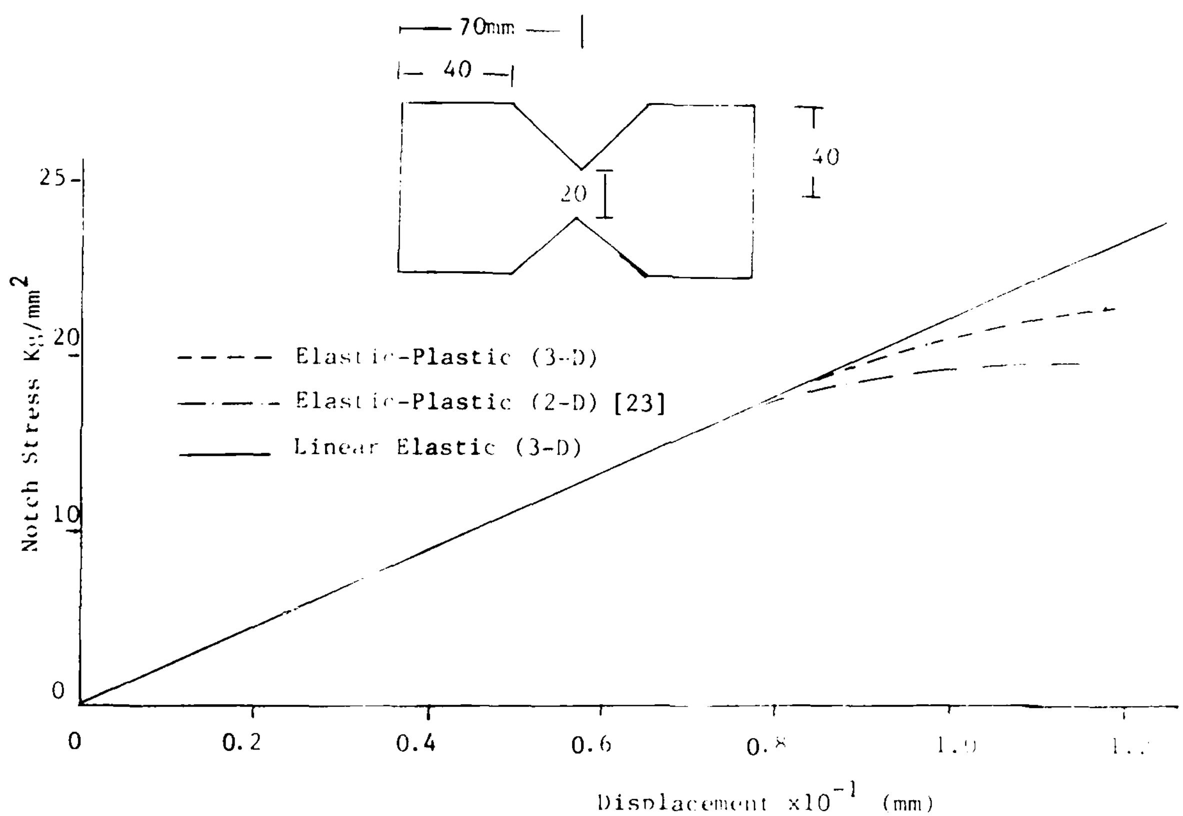


Figure 2. Notch Problems

A symmetric three-layer square laminate (0°/90°/0°) of dimensions a x a x h and subjected to uniformly distributed vertical load of intensity q_o on the top surface is analyzed. The plate is simply supported on all the edges. Taking advantage of the symmetry, only a quarter plate is analyzed.

The material of the lamina is assumed to have the following properties with respect to the material symmetry axes:

Linear Elastic Parameters

$$E_1 = 1.725 \times 10^8 \text{ kN/m}^2 \ (25 \times 10^6 \text{ psi})$$
$$E_2 = E_3 = 6.89 \times 10^6 \text{ kN/m}^2 \ (10^6 \text{ psi})$$
$$G_{12} = G_{13} = 3.45 \times 10^6 \text{ kN/m}^2 \ (0.5 \times 10^6 \text{ psi})$$
$$G_{23} = 13.78 \times 10^6 \text{ kN/m}^2 \ (0.2 \times 10^6 \text{ psi})$$
$$\nu_{12} = \nu_{31} = \nu_{32} = 0.25.$$

Parameters in the Ramberg-Osgood Model

	E_i	E_p	p	m
E_1	25×10^6 psi	5×10^6 psi	5×10^3 psi	2
	(1.726×10^8 kN/m^2)	(0.345×10^8 kN/m^2)	(0.345×10^5 kN/m^2)	
$E_2 = E_3$	1×10^6 psi	2×10^5 psi	5×10^3 psi	2
	(6.89×10^6 kN/m^2)	(1.392×10^6 kN/m^2)	(0.345×10^5 kN/m^2)	

Plasticity Parameters

Two different analyses are carried out here using 1) high strength model with the yield stresses X = 10,000 psi Y = 4000 psi and Z = 4000 psi and 2) low strength model with yield stresses X = 5000 psi, X = 2000 psi and X = 2000 psi.

Plots of the nondimensionalized deflection versus nondimensionalized load are presented in Figs. 3 - 5 for side-to-thickness ratios a/h = 5, 10, 20 and 100. The figures also contain results obtained by the use of the modified Ramberg-Osgood model in the two-dimensional laminated plate theory (see [5, 13]). For a/h ratios 5 and 10 (see Figs. 3 and 4), the results are in close agreement in the initial stages of the load, and the deflections predicted by the plate theory are lower than those predicted by three-dimensional theory in higher load levels. For (a/h) = 20 (see Fig. 5), the results obtained by the plate theory do not agree closely with those of the three-dimensional elasticity theory. This is due to the fact that full integration (2x2x2) is used in the 3-D analysis, which results in so-called locking of element stiffness coefficients [13].

The deflections obtained in three-dimensional analysis using the Ramberg-Osgood model and elastic plastic model are also compared in Fig. 4 for a/h = 10. A similar comparison is shown for (a/h) = 100 in Fig. 6. It is observed that the Ramberg-Osgood model is able to predict an average of the deflections produced by the plasticity models. Figures 6 and 7 contain similar results for stresses (σ_x) at the center of the plate. Here also, the Ramberg-Osgood model predicts an average of the stresses compared to the stresses predicted by the elastic-plastic models. This observation is encouraging to recommend the use of the Ramberg-Osgood model for design purposes because the Ramberg-Osgood model is much simpler than the elastic-plastic analysis which requires many iterations at each load level. These figures clearly demonstrate the effect of material nonlinearity on the behavior of a simply supported composite plate subjected to uniformly distributed load.

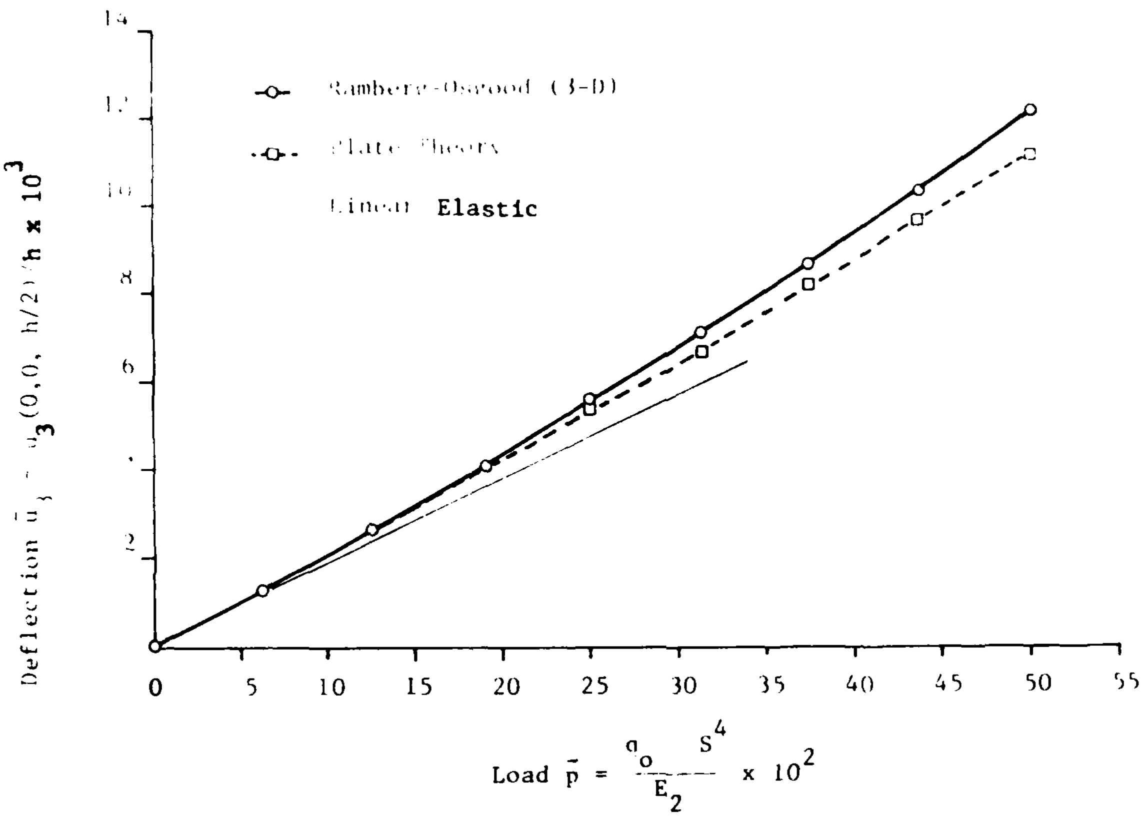


Figure 3. Load-Deflection Curve for Cross-Ply (0°/90°/0°, equal Thickness Layers) Square Plate Under Uniformly Distributed Load q_o (S = a/h = 5).

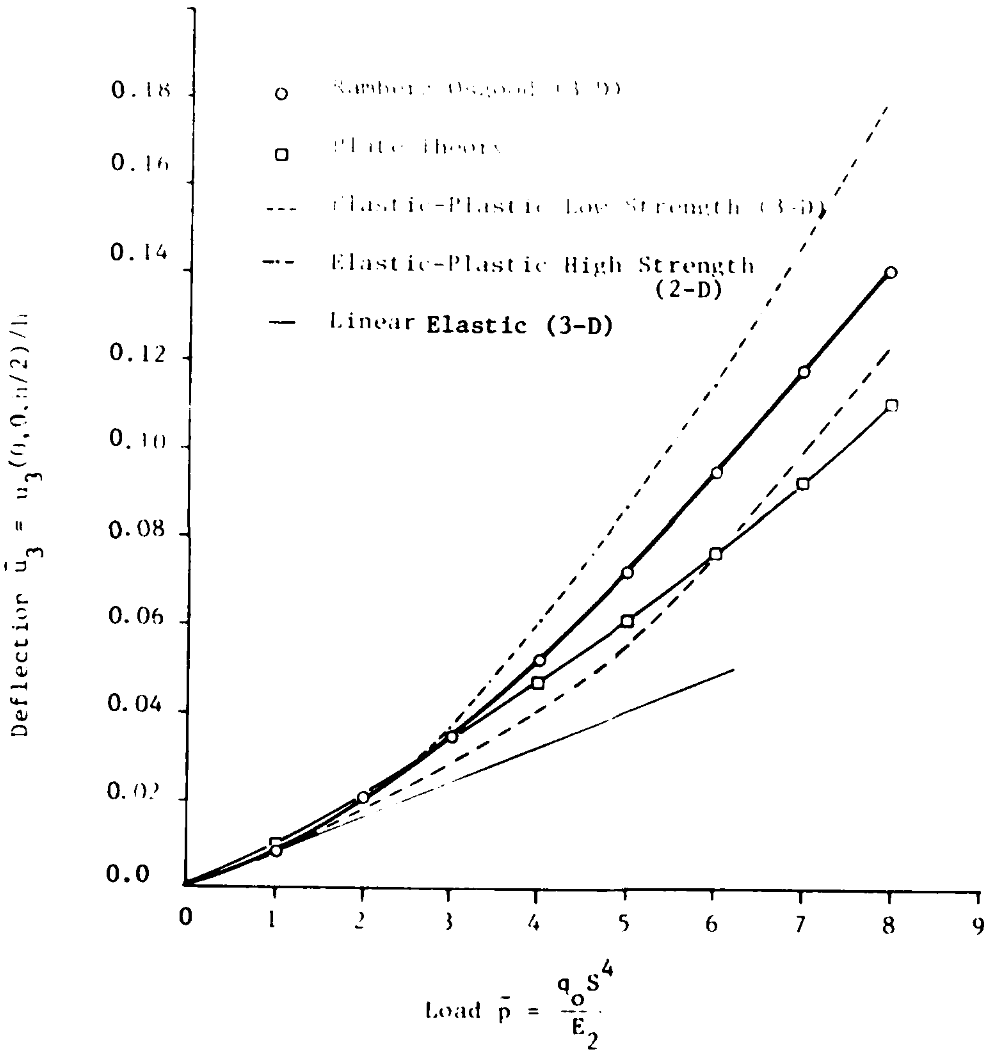


Figure 4. Load-Deflection Curve for Cross-Ply (0°/90°/0°, Equal thickness Layers) Square Plate Under Uniformly Distributed Load q_o ($S = a/h = 10$).

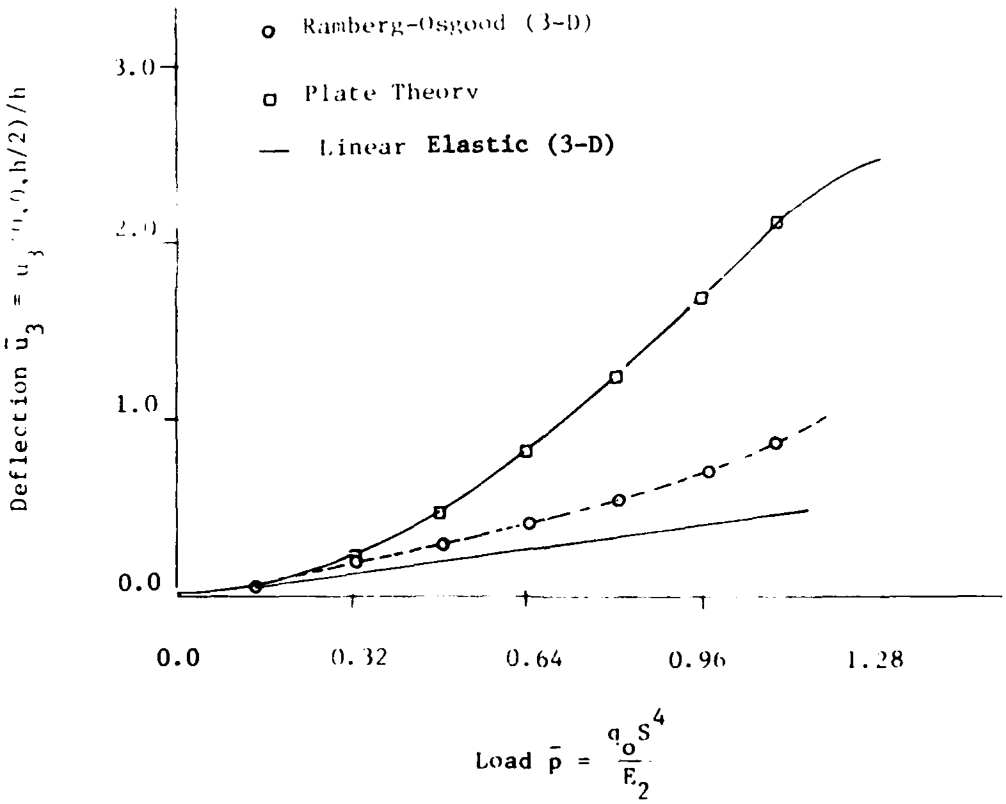
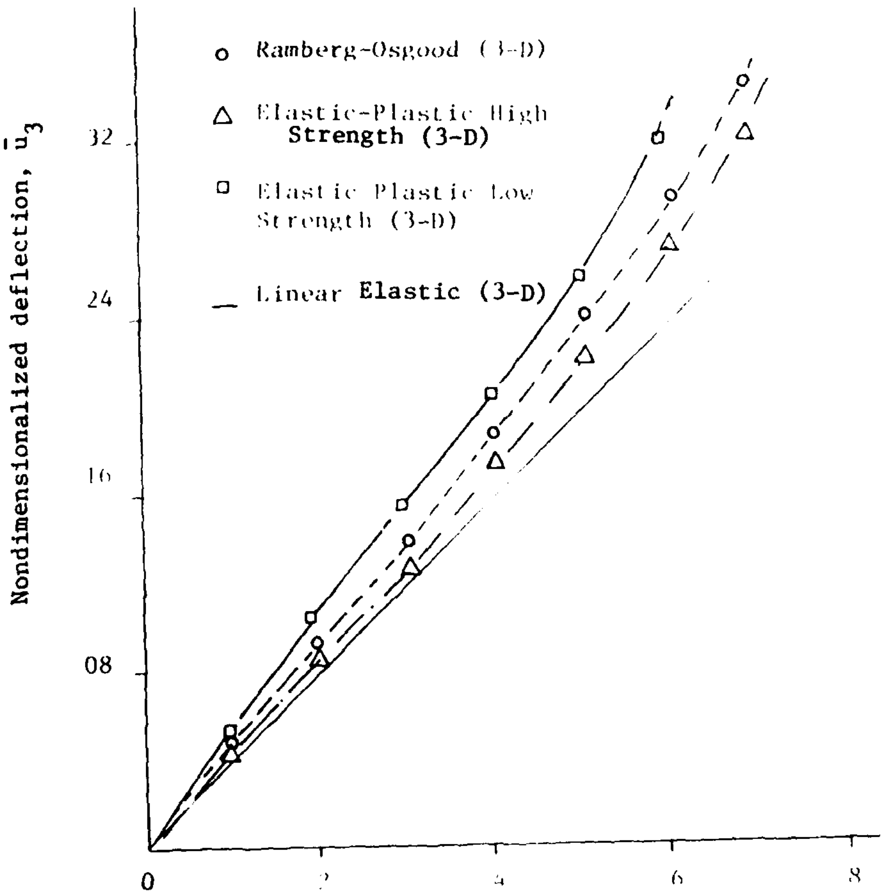


Figure 5. Load-Deflection Curve for Cross-Ply (0°/90°/0°, Equal Thickness Layers) Square Plate Under Uniformly Distributed Load q_o ($S = a/h = 20$).



Load $\bar{p} = \frac{q_o S^4}{E_2} \times 10^{-4}$

Figure 6. Load Deflection for Cross-Ply (0°/90°/0°, Equal Thickness Layers) Square Plate Under Uniformly Distributed Load q_o (S = a/h = 100).

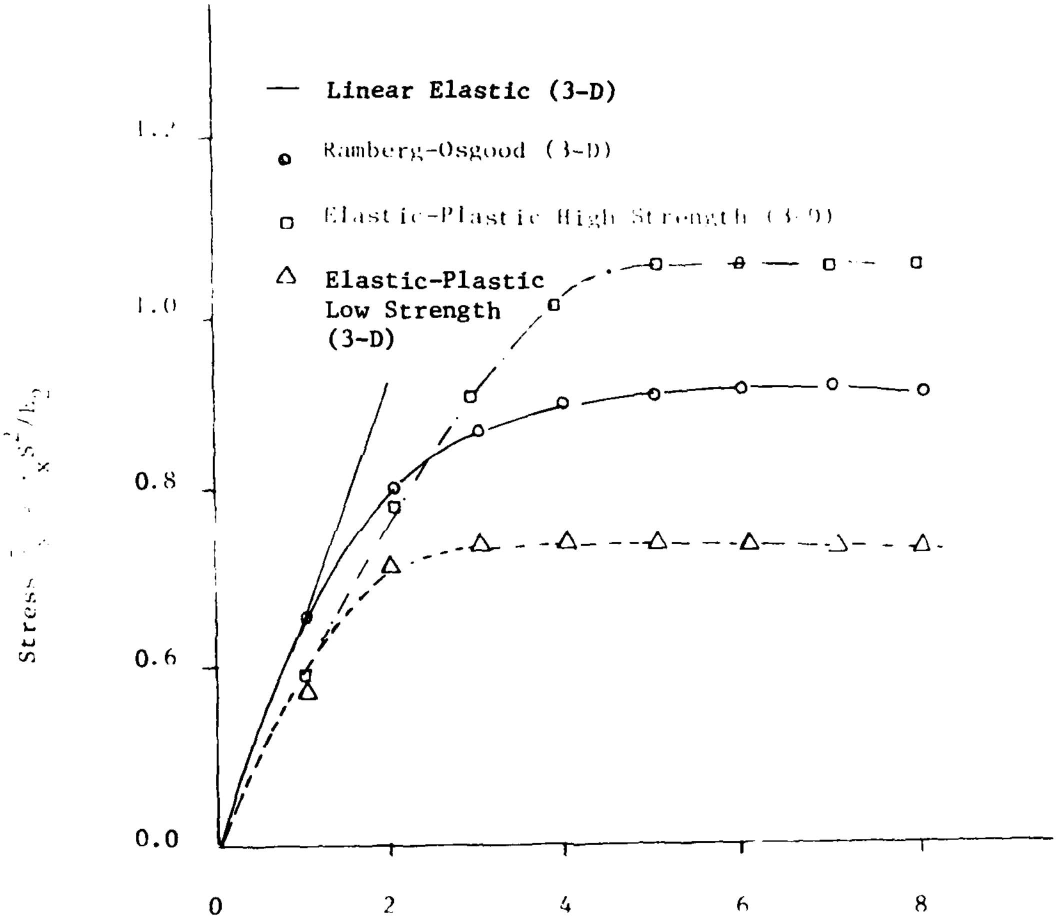


Figure 7. Normal Stress vs. Load for Cross-Ply (0°/90°/0°, Equal Thickness Layers) Square Plate Under Uniformly Distributed Load q_o ($S = a/h = 10$).

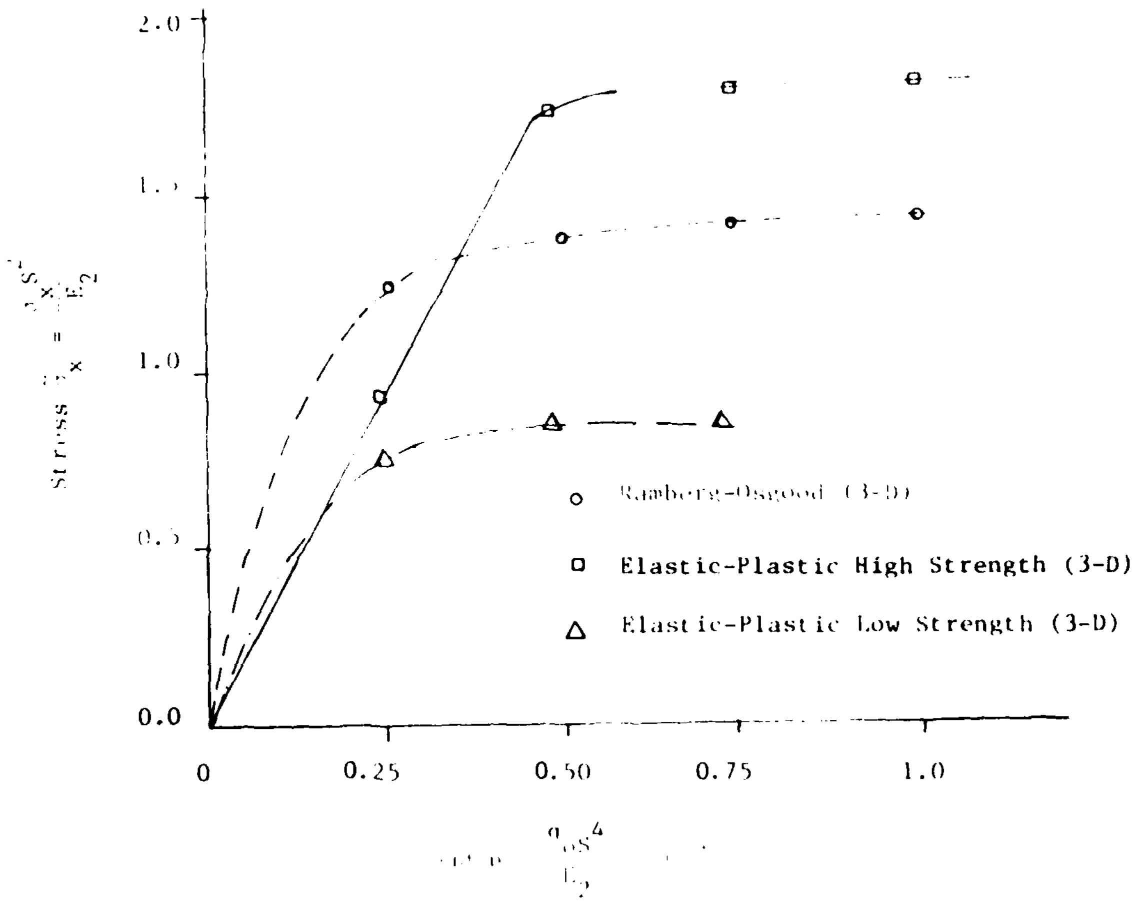


Figure 8. Normal Stress vs. Load for Cross Ply (0°/90°/0°, Equal Thickness Layers) Square Plate Under Uniformly Distributed Load q_o (S = a/h = 100).

CONCLUSIONS

A fully three-dimensional analysis of cross-ply laminated plates with geometric nonlinearity is presented. Two types of material nonlinear models are used: (1) uncoupled one-dimensional modified Ramberg-Osgood relation, and (2) elastic-plastic model with modified Hill's criterion for anisotropic media. It is observed that the uncoupled Ramberg-Osgood model was able to predict the average material behavior compared to the elastic-plastic model. The dflections obtained by the three-dimensional analysis using the uncoupled Ramberg-Osgood model are compared with those obtained by the shear deformation plate theory. Reasonable agreements are seen at lower load levels up to the plate side to thickness ratios a/h = 10.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The study deals with a fully three-dimensional analysis of laminated composite plates accounting for material nonlinearity. The finite-element method with trilinear interpolation of the three displacements is used to model layers of the laminate (both in the plane of laminae and through the thickness of the laminate). The modified Romberg-Osgood material constitutive relation is used to update the principal moduli in the plane of the laminate. The elastic-plastic model with Hill's criteria for anisotropic criteria is also used to analyze laminates. The shear deformable plate-theory element is also used in the study.

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